The Circle of Fifths as a Perceptual Model for Chordal Complexity

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I. INTRODUCTION

The following experiment tests the psychological validity of a perceptual model of harmonic complexity developed by Philippe Macnab-Séguin, based on the circle of fifths. This model hypothesizes that chords with notes spaced further apart on the circle of fifths will be perceived as more complex, whereas chords with notes closer together on the circle will be perceived as less complex, or simpler.

The model came about gradually through Macnab-Séguin's compositional work, by analyzing harmonic progressions that were arrived at intuitively and watching closely his own bodily reactions (such as slight increases in muscle tension, the subtle feeling of wanting to raise/lower the eyebrows or chest, etc.) to composed and improvised chords. The result is a system of composition that accounts for not only the complexity of individual chords, but the "sharpness/flatness" (roughly positive/negative valence) of chords, the harmonic distance (complexity applied to the realm of harmonic motion)[1] and "sharpwise/flatwise" motion (valence applied to the realm of harmonic motion). The details of the compositional system are outlined in Macnab-Séguin's M.Mus thesis (2017).

Although Hutchison and Knopoff's (1978) sensory dissonance model may prove to be a more accurate and insightful perceptual model of harmonic complexity from a scientific perspective, Macnab-Séguin's model may serve as a simpler and more intuitive way of calculating complexity, aimed at use in composition and analysis. As such, the circle of fifths is seen as a heuristic tool rather than as an accurate representation of psychological processes.

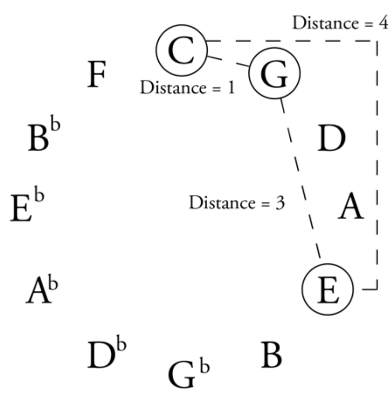

Figure 1. Distances of intervals in a C major chord.

Also, whereas the sensory dissonance model is aimed solely at calculating chordal complexity, the present study examines one part of a much larger model, leaving the notions of "sharpness/flatness" and harmonic motion for future research.

In our present model, chordal complexity is calculated by first determining the "distance" of each interval in the chord on the circle of fifths. The distance number of an interval is determined simply by counting the smallest possible number of steps between the two notes of the interval on the circle of fifths. For example, in a closed-voicing/root position major chord containing the pitches C-E-G, there are three intervals: C-E, C-G, and E-G. The distance between C and E, on the circle of fifths (see Fig. 1) is 4; the distance between C and G on the circle of fifths is 1, and the distance between E and G on the circle of fifths is 3.

Chordal complexity is then determined by calculating the average distance of each interval contained it the chord, while according more perceptual weight to notes lower in the chord and notes that are closer together in the voicing. Distances between the bass note and each upper note are accorded twice the perceptual weight as those from the second lowest note; distances from the second lowest note to each of the upper notes are accorded twice the perceptual weight as those from the third lowest note, and so on. This is called the "lower-note weighting". Likewise, distances between adjacent notes are given twice the perceptual weight as those between notes separated by one intervening note; distances between notes separated by one intervening note are given twice the perceptual weight as those between notes separated by two intervening notes, and so on.

[1] See also Lerdahl, 1988 and Milne, Laney and Sharp, 2016 for related models for testing harmonic distance.

This is called the "proximity weighting". A short mathematical example will suffice to explain how these perceptual weights are calculated:

> If we are given a set of 3 numbers, for example, [2, 3, 7], and we want to find the "weighted average" when 2 is given twice the "weight" of 3 and 3 is given twice the "weight" of 7, then it is as if we are actually calculating the average of the set [2, 2, 2, 2, 3, 3, 7]. Whereas the "unweighted" set's average would have been (2 + 3 + 7)/3 = 4, the "weighted" average is now: (2x4 + 3x2 + 7)/7 = 3.

These perceptual weights allow us to take the voicing of the chord into account. In Figure 2, both chords contain exactly the same pitch classes, but the first chord is voiced in a manner by which more complex distances occur between lower notes and adjacent notes, resulting in a higher complexity rating.

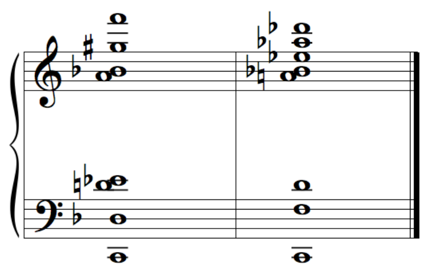

Figure 2. Two chords with complexity ratings of 6.516 (left) and 5.099 (right).

Finally, in order to compare the relative complexity of chords with varying numbers of notes, it is necessary to multiply this final average by a factor corresponding to the following formula, where n=the number of notes in the chord:

$$\frac{2^{n-1}-1}{2^{n-2}}$$

This means that a 3-note chord will be multiplied by a factor of 3/2, a 4-note chord by 7/4, a 5-note chord by 15/8, and so on. Without this factor, the results would be skewed, with chords containing less notes receiving calculations that are more complex. A very simple 4-note chord will suffice as an example for calculating complexity: C-E-G-Bb, as voiced in Fig. 3. Table 1 illustrates the calculation.

Intervals	C-E	C-G	C-Bb	E-G	E-Bb	G-Bb	
Distances	4	1	2	3	6	3	
Lower-note weighting	4*4=16	4*1=4	4*2=8	2*3=6	2*6=12	1*3=3	
Proximity weighting	4*4=16	2*1=2	1*2=2	4*3=12	2*6=12	4*3=12	
Totals	**32**	**6**	**10**	**18**	**24**	**15**	**= 105**

Weighted avg: 105/34=3.088 3.088*7/4 = 5.404

Table 1. Calculations for the chord illustrated in Figure 3.

Figure 3

The aim of this experimental study is to evaluate the validity of this perceptual model of harmonic complexity by asking participants to respond to single-chord stimuli using the descriptors "more complex" and "less complex." Participants will rate each chord's complexity using a sliding scale, and these responses will be compared to the chordal complexity rating of each chord, calculated using the model above. We anticipate that participant's ratings will increase along with the calculated chordal complexities across the range of stimuli. Such correlation would indicate that listeners' personal concept of "complexity" matches the concept presented with this model. Proof of this correlation would allow for future studies which would examine the perception of chordal complexity more closely.

II. METHOD

A. Subjects Six participants were recruited from a selection of McGill University graduate students, with a mean age of 26. All were considered musicians with a mean 15 years of formal training, and two participants reported having absolute pitch (AP).

B. Stimuli We created 50 chord stimuli using Native Instruments' "The Grandeur" sampled piano sound. Chords were composed with a specific voicing, in order to avoid implying a relationship between spacing and complexity (see Table 2). In addition, the bass note (voice 1) remained constant in order to avoid implying any tonal context or harmonic motion between stimuli. Since Macnab-Séguin's model includes a theory of "sharpness" and "flatness"[2] (Macnab-Séguin 2016), this factor was also controlled for by making an equal number of sharp and flat chords across the entire range of complexity. This was accomplished by creating a pair of chords, one sharp and one flat, within the complexity range of 2 to 2.2, a second pair within the range of 2.2 to 2.4, a third within the range of 2.4 to 2.6 etc. The total selection of chord stimuli ranges between 2 and 7 in complexity. For the task, each chord was presented three times, resulting in 150 randomized stimuli for each participant.

Voice 8	B♭4 to D5
Voice 7	A♭4 to C5
Voice 6	F4 to A4
Voice 5	E♭4 to G4
Voice 4	B♭3 to E♭4
Voice 3	E3 to A3
Voice 2	A2 to E♭3
Voice 1	Always C2

Table 2. Voicing of stimuli. C4 = middle C

C. Task The rating of stimulus task was presented in a standalone program designed by Yaolong Ju. The participants operated a horizontal, mouse-controlled slider whose position was displayed in a GUI window on their computer. A button labeled "Chord" would play one stimuli at a time when pressed. Participants were asked to rate the complexity of each stimulus by choosing a number rating along the scale from 1 to 7, with 1 = least complex and 7 = most complex. Stimuli were presented to each participant in random order, and each stimulus could be played once. After a complexity rating was submitted, the software would pause for 5 seconds, in order to reduce the possibility of persisting pitch memory or tonal context between stimulus (Farbood 2016: 72) With this timing constraint in place, the total experiment duration would be approximately 45 minutes.

D. Design Once we have established a basic correlation between complexity calculated with this model and listeners' ratings, a follow-up study could present stimuli using a comparison model, and would ask participants to choose the more-complex chord in a pair of closely-rated stimuli. This would further confirm a perceptual correlation to choral complexity calculated with this model. We chose a rating model for the current experiment, however, in order to determine if listeners *instinctively* rate chords with a higher complexity rating as more complex, and chords with a lower complexity rating as less complex. By foregoing any training period of the complexity model or stimuli, we allow for a subjective "complexity" response from each participant. In general, we expect listeners' complexity ratings to mirror the complexity calculations of the stimuli.

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[2] For a discussion of sharpness/flatness calculations and how this relates to complexity, see Macnab-Séguin 2016, p. 2-11.

E. Analysis The participants' responses were recorded by the program as a text file, with their numerical complexity rating value assigned to each chord stimulus file name. The first ten responses were omitted from the final results, in order to allow for any experimental error resulting from participants becoming accustomed to the program design, stimulus type, audio volume, etc. Results were then sorted in order of stimulus complexity calculation, and the mean response to each chord calculated for each participant individually. These responses were represented on a scatter plot, and the trends of responses were compared using a linear regression for each participant.

III. Results

As shown in Figure 4, participants' ratings generally trend upward as complexity calculation increases, with a correlation coefficient of 0.8735. In comparison, the correlation coefficient for the participants' ratings and the

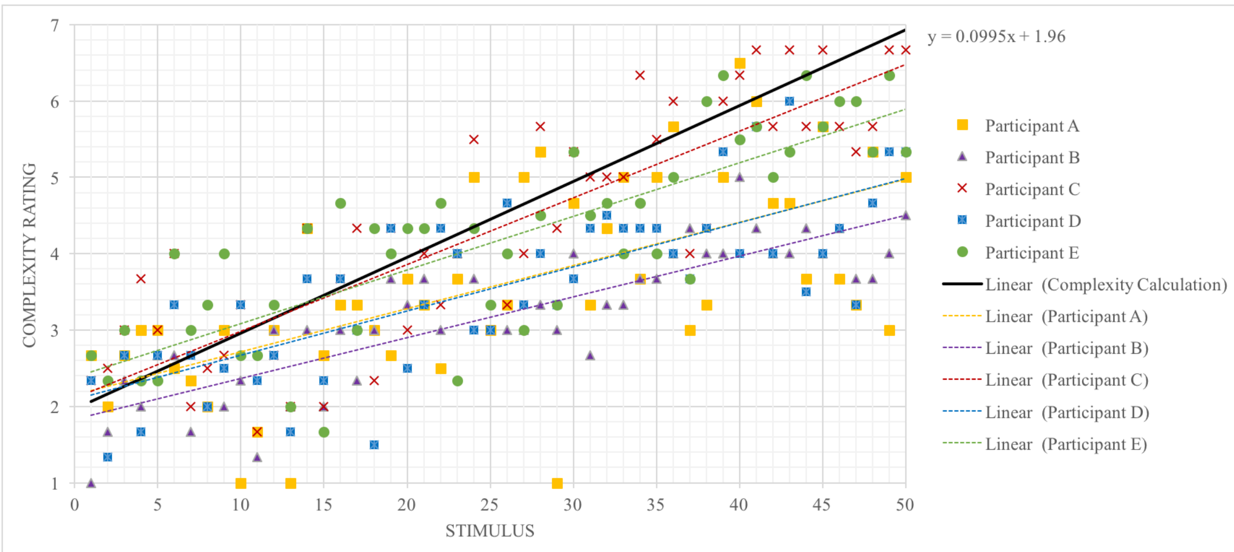

Figure 4. Scatter plot of participant responses (averaged) for each stimulus, including trend lines for each participant.

number of pitch classes in the chord was 0.7346. The relatively similar slopes of each linear regression suggest a correlation between chordal complexity calculated with the present model and each participants' response rating. However, it is clear that the compass of responses occupies a much wider range as stimulus complexity increases. This could be explained by the different upper and lower perceptual thresholds for complexity assigned by each participant. Inclusion of a training segment during the experiment could lead to a more accurate response to stimuli on the highest and lowest ends of the complexity scale.

Participants C and E show the narrowest response compass as the complexity of stimuli increases. However, an analysis of the standard deviation of responses over the course of the experiment does not suggest that these participants chose ratings with any greater degree of accuracy as the experiment progressed.

IV. Discussion

The correlation coefficient of 0.8735 between the present model and participants' ratings seems to suggest that the model can be used as a fairly accurate heuristic for predicting the perceived complexity of a given chord. Since the principles behind the model are fairly simple and can be used intuitively, without the need for calculations, it may be of use for composers, either as foundation for creating a harmonic system or as a tool for analyzing and objectively evaluating their own music in the editing process. Knowledge of the "lower note" and "proximity" perceptual weights can also be very useful as a creative tool, as it allows composers to re-voice chords with the specific intention of increasing or decreasing perceived complexity.

Two interesting patterns in the data should be noted. The first manifests as a slight correlation between the "flatness" of the stimuli and their perceived complexity; in other words, flatness seems to contribute to the perception of complexity. The concepts of sharpness and flatness are presented and elaborated in Macnab-Séguin's *The Circle of Fifths as a Perceptual Model for the Analysis of Spectral Music* (2016), which describes the notions of sharpness and flatness as roughly translating to positive and negative valence, respectively. The measure of sharpness/flatness of a chord (hereafter S/F) is calculated in the same way as complexity, however *directionality* of the distance between notes on the circle of fifths is also taken into account: when the top note is *anticlockwise* on the circle in relation to the bottom note, the number used in the calculation is negative, whereas if the top note is clockwise on the circle in relation to the bottom note, is it used in the calculation as a positive integer. For example, even though a minor third and major sixth have the same "complexity" (3) a minor third has an S/F of -3, whereas a major sixth has an S/F of +3.

The correlation between S/F and participants' ratings of complexity was -0.2125, and flat chords (chords with a negative S/F) had an average rating of 3.89, whereas sharp chords had an average rating of 3.66. This could be due to two reasons. Firstly, circle of fifths *approximates* harmonic complexity of interval ratios, but flat intervals generally have more complex interval ratios than sharp ones (e.g., P5 is 2:3 whereas P4 is 3:4; a M2 is 8:9 whereas an equally-tempered m7 is approximately 9:16, etc.). Secondly, "often the concepts of flatness and complexity are conflated aurally, due to the fact that complexity and flatness both contribute to the overall impression of 'darkness'" (Macnab-Séguin, 2016).

The second pattern in the data that is interesting to note is the tendency for there to be a higher standard deviation in participants' ratings at higher levels of complexity, both within ratings of a single participant and between participants. There are a number of possible explanations for this phenomenon. One simple explanation could be that, in general, people are less used to hearing extremely complex chords in music, and thus are less likely to have developed mental models for their comparison. In addition, variation among participants' ratings could be accounted for by the fact that, given the voicing constraints of the stimuli, there are less possible options for creating very simple chords (one is constrained to a limited number of intervals), whereas there can be much greater differences between more complex chords (since more possible intervals are available). One result of this is that there is a greater possibility for *internal variation* in complex chords. For example, the middle register of a complex chord can actually be very simple if taken in isolation. This means that if a listener attends to different parts of the chord on different presentations of the stimuli, or if two different sets of speakers/earphones emphasize different frequency ranges, this will likely affect their perceived complexity.

The results could very likely be improved through improvements of the perceptual model itself. One obvious place for improvement is in the perceptual weights. First of all, the calculations do not technically take register into account, only the *order of the notes* from bottom to top. This means that the chord C4-D4-E4 would have an identical complexity calculation as C2-D4-E6. This would require a new formula which takes absolute pitch register into account. In addition, the logarithmic curve of perceptual weights may be skewed; in the creation of the stimuli, it was observed that the interval between the bass and second-lowest note had a greater effect on perceived complexity than was apparent in the calculations. Finally, in the future a new formula could be devised that accounted for the fact that flat intervals tend to be perceived as slightly more complex than sharp intervals.

Due to the low CPU demands necessary to make these calculations, and its possibility for extension into the realms of sharpness/flatness, harmonic distance (complexity applied to harmonic motion) and sharpwise/flatwise motion (valence applied to the realm of harmonic motion), it may be hoped that this model, with further refinements and perceptual experiments, could be applied to the analysis of databases of midi files. The fact that it is, in principle, applicable to any style of music (which remains to be evaluated), could also open up the doors to comparisons of harmonic syntax across vast stylistic and historical boundaries.

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